Trees, Fundamental Circuits & Cut-Sets ①

Tree → Connected graph without any circuit.

Null tree → without any vertex

— tree has to be a simple graph else there will be circuit

e.g. - rivers and its tributaries
- sorting of mail according to zip code

Properties →

Th^m → There is one and only one path b/w every pair of vertices in a tree

Proof → T → connected

∴ there must ~~be~~ exist at least one path b/w every pair of vertices in T.

Now suppose that b/w two vertices 'a' & 'b' of T, there are two distinct paths. Then union of these two paths will contain a circuit and T can not be tree. Hence the theorem.

Th^m → If in a graph G, there is one and only one path b/w every pair of vertices, then G is a tree

Proof → Existence of a path between every pair of vertices assures that G is connected. A circuit in a graph implies that there is at least one pair of vertices a, b s.t. there are two distinct paths b/w a & b. Since G has one and only one path b/w every pair of vertices, G can have no circuit. ∴ G is a tree.

~~Rule~~ Note→ A tree with n vertices has (n-1) edges

Note→ Any connected graph with n vertices and (n-1) edges is a tree

Note→ A graph G with n vertices, (n-1) edges and no circuits is connected.

A graph G with n vertices is called a tree if

1. G is connected and is circuitless or
2. G is connected and has (n-1) edges or
3. G is circuitless and has (n-1) edges or
4. There is exactly one path b/w every pair of vertices in G or
5. G is minimally connected graph.

Minimally Connected Graph→

A graph is said to be minimally connected graph if removal of any edge from it disconnects the graph.

A min. conn. graph can not have circuit otherwise we could remove one of the edges in the circuit and still leave the graph connected.

Note→ A graph is a tree iff it is minimally connected.

Th. In any tree (with two or more vertices), there are at least two pendant vertices.

Proof. Let G be any tree having n vertices. Then G has (n-1) edges. Since each edge contributes two degrees, the sum of the degrees of all vertices in G is 2(n-1). Now 2(n-1) degree are to be divided among n vertices in G. Let the number of vertices of degree one in G be x. Since no vertex in a tree can be of zero degree, we have—

$$\frac{2(n-1)-x}{n-x} \geq 2 \Rightarrow x \geq 2$$

Distance and Centres in a Tree →

In a connected graph G, the distance $d(v_i, v_j)$ b/w two of its vertices v_i and v_j is the length of the shortest path (i.e. the number of edges in the shortest path) b/w them.

Note The distance b/w vertices of a connected graph is a metric.

(i) non-negativity i.e. $d(x,y) \geq 0$, $d(x,y) = 0 \Leftrightarrow x = y$

(ii) $d(x,y) = d(y,x)$

(iii) $d(x,y) \leq d(x,z) + d(z,x)$ for any z

Note In a tree, vertex of degree 1 is called leaf

Scanned by CamScanner

Eccentricity of a vertex →

(2a)

'v' in a graph G is the distance from v to the vertex farthest from v in G i.e.

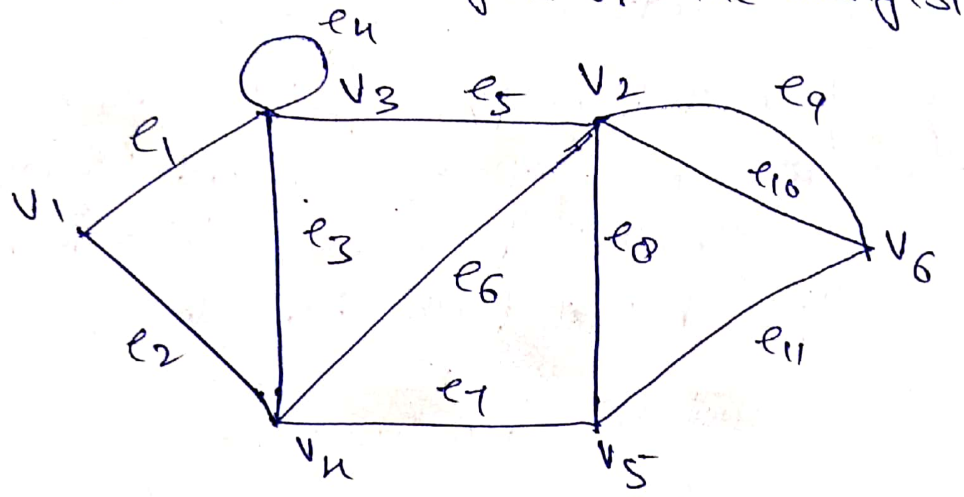
$$E(v) = \max_{v_i \in G} d(v, v_i)$$

Centre → A vertex with minimum eccentricity in a graph G is called a centre of G.

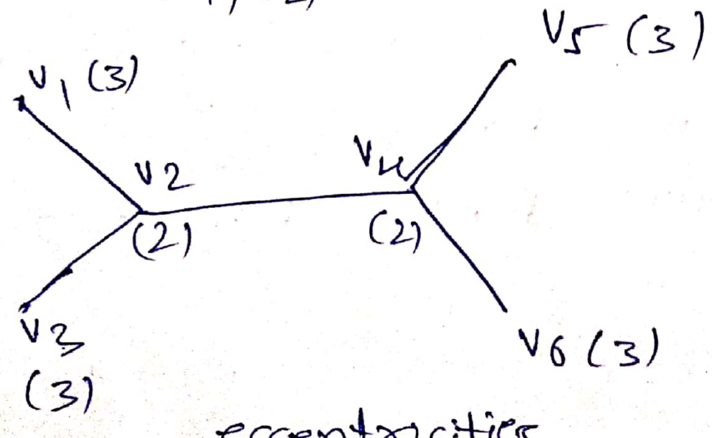
Note → A tree has either one or two centres

Radius → The ecc. of a centre in a tree

Diameter → The length of the longest path in T.



$d(V_1, V_2) = 2$



Centre = V_2, V_4

radius = 2

diameter = 3

eccentricities